AP CALCULUS AB

Name:

SUMMER PACKET 2024

Dear AP Calculus AB student,

Welcome! Calculus AB is both a challenging and rewarding course, and we're excited that you've decided to take this class. To help you succeed in this course and prepare you for the rigorous expectations next fall, you should feel very confident and independent in the topics listed below. Please read through the list carefully and make sure you completely understand each topic.

*__Linear Equations:__ Students should be able to graph linear equations, find and represent equations of lines in a variety of formats (point-slope, slope-intercept, etc.), understand the slope relationship between parallel/perpendicular lines, and draw both tangent and normal lines to a graph at a given point.

*__Factoring:__ It is absolutely imperative that students factor efficiently in a variety of methods, such as grouping, GCF, sum/difference of squares, and sum/product. In addition, students should be algebraically proficient at completing the square.

*__Basic Function behavior and notation:__ Students should be able to graph and model the behavior of quadratic, cubic, and other polynomial functions, as well as the functions $f(x)=|x|$, $f(x)=\sqrt{x}$, and any rational function. In addition, students should be able to work fluently with function notation, find domain restrictions, perform compositions, and write solutions in interval notation.

*__Polynomial and Rational Functions:__ Students should be able to analyze the behavior of both polynomial and rational functions, including finding *x*/*y*-intercepts, writing equations of horizontal and vertical asymptotes, stating domain restrictions, and finding any points of discontinuity (i.e. a hole in the graph). Students should also know algebraically the difference between a rational function having a hole versus a vertical asymptote.

*__Exponential and Logarithmic behavior:__ Students should be able to sketch the graphs (and transformations) of the functions $f(x)=a^x$ and $f(x)=\log_b(x)$, which includes asymptotes and intercepts. In addition, students should be able to evaluate, simplify, and justify equivalencies of basic logarithmic and exponential expressions without a calculator. Finally, students should be able to work fluently between exponential and logarithmic functions as inverse relationships.

*__Trigonometry:__ Students should be able to sketch the graphs of $f(x)=\sin(x)$ and $f(x)=\cos(x)$, with our without basic transformations. Students should also know the behavior of the 4 remaining trig functions. Students must be able to quickly calculate the trigonometric ratios of 30º-60º -90º, 45º-45º-90º, and quadrantal angles. Finally, students must know the 6 inverse-trigonometric functions and clearly know the restricted range for each.

*__Graphing Calculator Skills:__ Students should know how to calculate points of intersection between 2 graphs, find zeros of a function, and find max/min values.

THIS PACKET IS DUE ON __MONDAY, AUGUST 19TH 2024.__

We look forward to meeting you in August!

Sincerely,

The AP Calculus AB Team: Mrs. Zinzer, Mrs. Murphy, Mr. Kim

AP CALCULUS AB

Part I: General function knowledge.

1.) Let $f(x) = 3x^2 - 4x + 5$. Find the following.

a.) $f(-3)$ **b.)** $f\left(\frac{2}{5}\right)$

c.) $f(-2x)$ **d.)** $f(x+h)$

2.) Let $f(x) = \frac{1}{\sqrt{x}}$. Find AND SIMPLIFY the following.

a.) $f(12)$ **b.)** $f\left(\frac{9}{25}\right)$

c.) $f\left(e^{6x}\right)$ **d.)** $f\left(x^{-2}\right)$

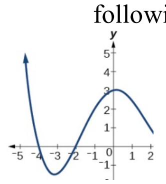
3.) Let $g(x) = 3x^2$.

a.) Find $g(x+h)$

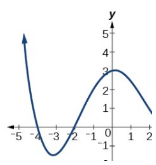
b.) Simplify the expression $\frac{g(x+h)-g(x)}{h}$

4.) Let $g(x) = -2x^2 + 4x$. Find and simplify the expression $\frac{g(x+h)-g(x)}{h}$

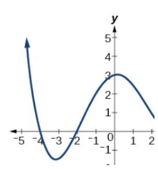
5.) Use the graph of the polynomial $m(x)$ for the following questions.



a.) Draw the line that is tangent to the graph of $m(x)$ at $x = 2$. What is the approximate slope of this tangent line?



b.) A NORMAL line is defined to be perpendicular to a tangent line. Draw the tangent line AND the normal line at $x = -1$.



c.) Draw the secant line connecting the points on $m(x)$ from $x = -2$ to $x = 2$. Approximate the average rate of change of $m(x)$ over this interval.

d.) Is there any point on the graph of $m(x)$ that seems to have a horizontal tangent line? Where is this?

6.) Let $f(x)$ be a polynomial function with select values given below. The table also includes the slope values on $f(x)$ at each point.

x	0	1	2	3
$f(x)$	4	–3	–1	2
Slope	–3	2	4	–1

a.) Use the table above to find the equation of the line drawn when $x = 2$. Write your equation in point-slope form, $(y - y_1) = m(x - x_1)$.

b.) Use your result in part **a** to find the y value of the line when $x = 2.2$.

c.) Calculate the average rate of change of $f(x)$ between $x = 3$ and $x = 1$.

For the next section, reminder as an example, since $\sin\frac{\pi}{6} = \frac{1}{2}$, this means $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.
Note: $\sin^{-1}(x) = \arcsin(x)$. These notations are equivalent.

Part II: Trigonometry

Directions: For problems #7–26, evaluate each expression.

7.) $\cos\left(\frac{2\pi}{3}\right) =$ _____

17.) $\sin\left(\frac{3\pi}{2}\right) =$ _____

8.) $\cos(0) =$ _____

18.) $\tan(0) =$ _____

9.) $\sin\left(-\frac{\pi}{4}\right) =$ _____

19.) $\cos\left(\frac{\pi}{2}\right) =$ _____

10.) $\csc\left(\frac{\pi}{6}\right) =$ _____

20.) $\csc\left(\frac{4\pi}{3}\right) =$ _____

11.) $\sec\left(\frac{3\pi}{2}\right) =$ _____

21.) $\sin\left(\frac{5\pi}{6}\right) =$ _____

12.) $\cos(\pi) =$ _____

22.) $\cos\left(\frac{3\pi}{4}\right) =$ _____

13.) $\cot\left(-\frac{\pi}{6}\right) =$

20.) $\arcsin(-1) =$ _____

14.) $\arctan(-1) =$ _____

22.) $\arcsin\left(-\frac{\sqrt{3}}{2}\right) =$ _

15.) $\arccos\left(-\frac{1}{2}\right)$

24.) $\text{arc}\cot(-1) =$ ____

16.) $\arccos(0) =$ _____

26.) $\arccos(1) =$ _____

27.) Sketch and label 2 periods of each function below:

a.) $f(x) = 4\sin x$

b.) $f(x) = -\cos x + 1$

c.) $f(x) = \sec x$

d.) $f(x) = \tan x$

28.) Solve $(1 + 2\sin\theta)(\cos\theta - 1) = 0$ over the interval $[0, 2\pi)$

29.) Solve $\tan(\theta) = -1$ over the interval $[0, 2\pi)$

30.) Solve $4 - 2\sec\theta = 8$ over the interval $[0, 2\pi)$

Part III: Polynomial and Rational Functions

31.) Factor a GCF of the following expressions:

a.) $4x^2y^6z - 10x^4y^2z^3$

b.) $27x^3(x+1)^5(2x-5)^2 + 15x^5(x+1)^4(2x-5)^3$

32.) Find the real solutions for the following equations.

a.) $2x^2 - x - 15 = 0$

b.) $3x^2 + 2x - 8 = 0$

c.) $9x^2 + 27x = 0$

d.) $3x^3 + 4x^2 = 7x$

e.) $6x^2 + x - 12 = 0$

f.) $-x^2 + 10x - 16 = 0$

33.) Find ALL solutions (real and non-real) to each equation

a.) $2x^3 + 3x^2 + 8x + 12 = 0$

b.) $x^4 - 1 = 0$

34.) For the function $f(x) = 3x^2(x-4)^3(x+1)$,

a.) Determine when $f(x) = 0$.

b.) Using a sign chart, determine when $f(x) < 0$.

Directions: The following rational functions each have a hole at the given x-value. Find the corresponding y value of this point of discontinuity.

35.) $f(x)=\dfrac{2x-6}{x^2+3x-18}$ at $x=3$ ____________

36.) $F(x)=\dfrac{2x^2+6x}{18x}$ at $x = 0$ ____________

37.) $t(x)=\dfrac{\sqrt{x}-2}{x-4}$ at $x = 4$ ____________

38.) $t(x)=\dfrac{\sqrt{x+5}-\sqrt{5}}{x}$ at $x = 0$ ____________

39.) $k(x)=\dfrac{\frac{1}{x+3}-\frac{1}{6}}{x-3}$ at $x = 3$ ____________

40.) $W(x)=\dfrac{x-5}{\sqrt{2x-1}-3}$ at $x=5$ ____________

41.) Each of the following rational functions have both vertical AND horizontal asymptotes. Write the <u>equations</u> of all asymptotes.

a.) $f(x)=\dfrac{2x-1}{x^2-x-6}$ ____________

b.) $f(x)=\dfrac{5x+1}{3-x}$ ____________

c.) $f(x)=\dfrac{(4x-1)^2}{(2x+5)(3x-7)}$ ____________

42.) For $f(x) = 4(x-1)^3(x+3)^5 + 7(x-1)^4(x+3)^4$,

a.) Factor $f(x)$. (Hint: use a GCF. Hint: look at problem 31a&b.)

b.) Determine when $f(x) = 0$.

c.) Use a sign chart to determine the interval(s) when $f(x) > 0$.

43.) Suppose $f(x) = \dfrac{3x^2(x^2+16)^2 - 4x^4(x^2+16)}{(x^2+16)^4}$

a.) Find all real values of x such that $f(x) = 0$.

b.) Using a sign chart, find all intervals for which $f(x) > 0$.

44.) Rewrite each expression by completing the square.

a.) $f(x) = x^2 - 8x + 18$

b.) $f(x) = 2x^2 - 12x + 21$

c.) $f(x) = \dfrac{1}{4}x^2 + x + 1$

d.) Complete the square in the denominator:

$$f(x) = \frac{1}{x^2 + 18x + 75}$$

Part IV: Logs and exponents

45.) Fill in the boxes so the following equations are true.

a.) $\sqrt{x} = x^{\square}$

b.) $x\sqrt{x} = x^{\square}$

c.) $x^5 \cdot x^{\square} = \dfrac{1}{x^{15}}$

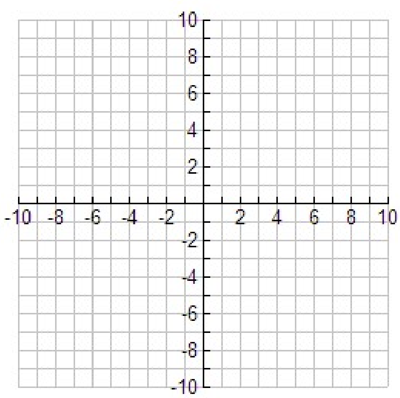
d.) $\dfrac{1}{x^3} = x^{\square}$

e.) $\sqrt[4]{x^3} = x^{\square}$

f.) $x^5 \cdot x^{\square} = x^{15}$

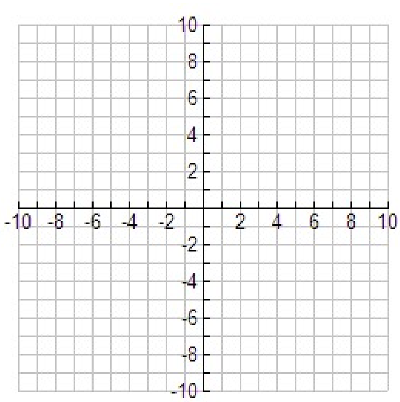
46.) Sketch the graph of each function below. Label at least 2 points and the equation of any asymptotes.

a.) $f(x) = e^x + 1$



Domain: Range:

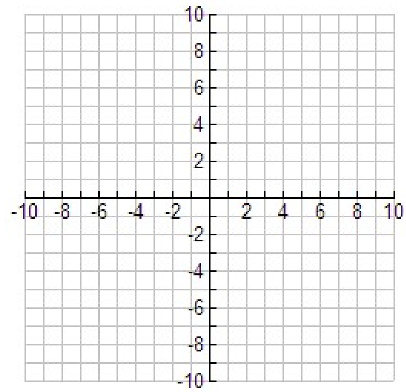
b.) $f(x) = 2\left(\dfrac{1}{3}\right)^x$



Domain: Range:

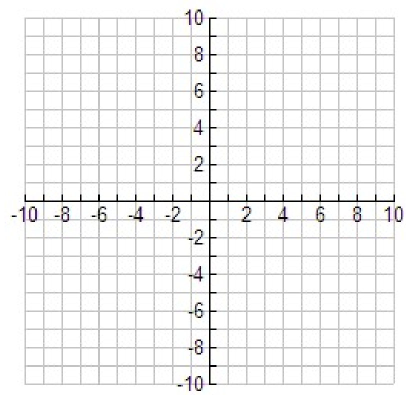
47.) Sketch the graph of each function below. Label at least 2 points and the equation of any asymptotes.

a.) $y = \ln x$



Domain: Range:

b.) $g(x) = \log_2(x-1)$



Domain: Range:

48.) Find x if $\log_3 x = 4$.

49.) Find x if $4\ln(x+2) = 24$.

50.) Fill in the box: $\log(2) + \log(12) = \log(\square)$

51.) Fill in the box: $\ln(32) = \ln(2^{\square}) = \square \cdot \ln(2)$

52.) Evaluate the following WITHOUT a calculator:

a.) $\log_6 6 =$ _______ **b.)** $\log_4 64 =$ _______

c.) $\ln 1 =$ _______ **d.)** $\ln\left(e^4\right) =$ _______

e.) $\log_2\left(\frac{1}{32}\right) =$ _______ **f.)** $\log\left(\frac{1}{10}\right) =$ _______

g.) $\log_7(0) =$ _______ **h.)** $\ln\sqrt{e} =$ _______

i.) $\log_3\left(\frac{1}{9}\right) =$ _______ **j.)** $\log_5 1 =$ _______

53.) Solve: $2e^{2x}x^6 - 6e^{2x}x^5 = 0$ (Hint: see problem 31)

54.) Solve: $e^x \cdot \frac{1}{x^4} - 4e^x \frac{1}{x^3} = 0$ (Hint: problem 31.)

55.) Fill in the blank:

a.) $\log(\square) = 3$

b.) $\ln(\square) = -2$

c.) $\ln(\square) = 1$

56.) Solve: $\frac{1 - \ln x}{x^2} = 0$

Part V: Calculator Proficiency (RADIAN MODE)

57.) CALCULATOR OK!

Let $f(x) = \sin(x) - 0.5x$ and let $g(x) = -3x^2 + 2x + 4$. Answer the following:

a.) How many times do the graphs of $f(x)$ and $g(x)$ intersect? Sketch a quick graph from your calculator to support your solution, including labeling the x and y axes to show your viewing screen.

b.) What are the zeros (x-intercepts) of $g(x)$? Sketch a quick graph from your calculator to support your solution, including labeling the x and y axes to show your viewing screen.

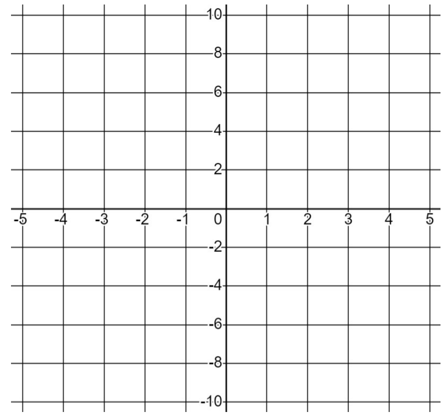
c.) Using your answer from part (b), find all intervals for which $g(x) > 0$.

d.) Find the coordinates of the maximum of $g(x)$.

58.) CALCULATOR OK!

Let $f(x) = 3\cos x$ and let $g(x) = [f(x)]^2 - 2$.

a.) Make a sketch of the graph of $g(x)$ over the interval [-5,5].



b.) What is the amplitude of $g(x)$?

c.) On the interval [-4,4], how many relative minimums does $g(x)$ have?

d.) Is $g(x) > 0$ or is $g(x) < 0$ at $x = 4$?

e.) Is $g(x)$ increasing or decreasing at $x = 4$?